

First-Order differential equation

A manufacturer has found that the change in the distribution cost C as sales V increase is equal to a constant a multiplied by sales plus another constant b . If $C = 0$ when $V = 0$, find C as a function of V .

Solution

We set up the differential equation:

$$C'V = aV + b$$

That is:

$$\begin{aligned}\frac{dC}{dV} &= aV + b \\ dC &= (aV + b)dV\end{aligned}$$

Integrating both sides:

$$C = \frac{aV^2}{2} + bV + K$$

Using the point:

$$0 = K$$

Thus:

$$C = \frac{aV^2}{2} + bV$$